

How rain starts

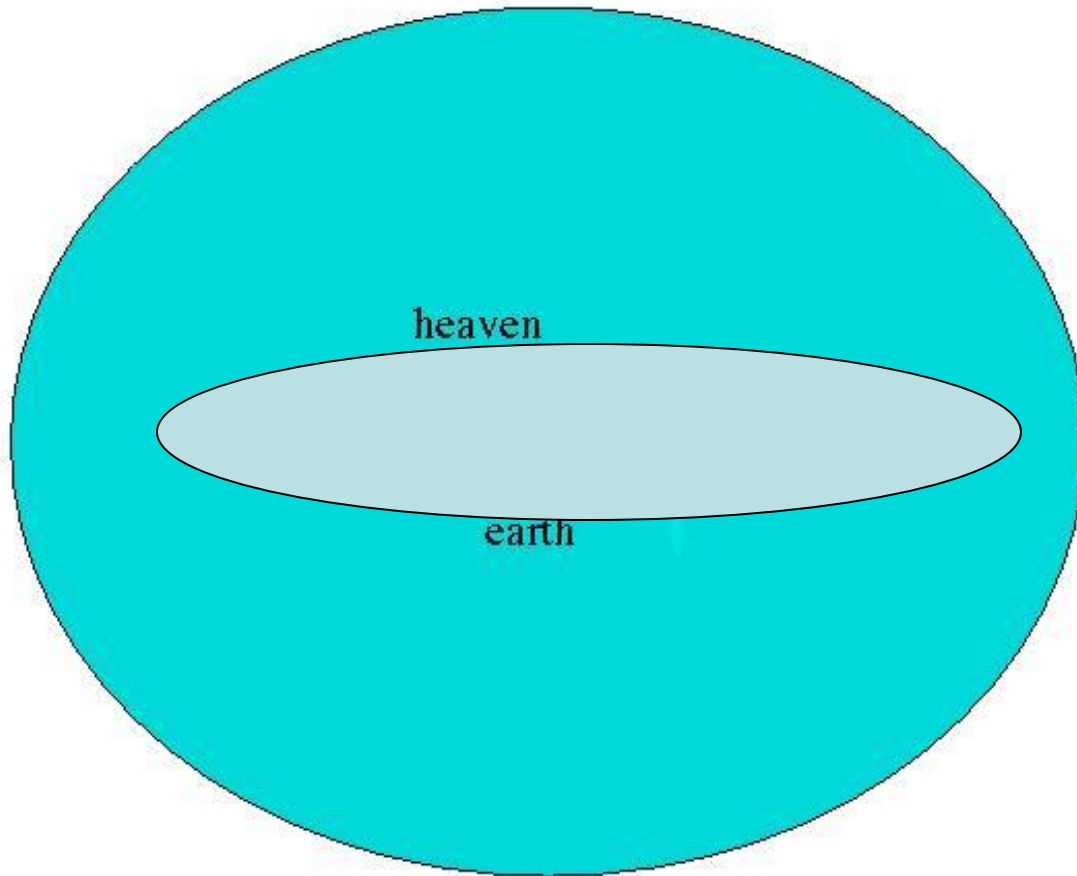
From primordial chaos
to modern turbulence

G. Falkovich
Weizmann Institute



Slaying the dragon of primordial chaos (Tiamat) by Enlil (Marduk, Ashur, St. George) allowed the creation of our orderly world out of chaos.

He sliced her in half like a fish for drying:
Half of her he put up to roof the sky,
Drew a bolt across and made a guard to hold it.
Her waters he arranged so they could not escape.
Enuma Elish



“... that call for the waters of the sea, and
pour them out upon the face of the earth.”

Amos, V8



Strepsiades: Do you think Zeus always rains new water down
or does the sun draw the old up to be re-used?

Amyntas: I don't know and I don't care.

Strepsiades: How do you expect to get your money if you
know nothing of meteorology?

Aristophanes, CLOUDS



“If the air is warm, these vapors rise... and the clouds collect one above the other... as if they were mountains of combed cotton. But if cold comes in from above, the vapors collect and become water; they become drops and fall from the upper region of the cloud. These little drops unite with one another until, if they come out of the lower boundary of the cloud, they are large drops of rain.”

Encyclopaedia written by “Brethren of Purity”
Basra, tenth century AD

Vapor is not air



The equalization of pressure caused the air in the unevacuated flask to “give up its surplus moisture”.

Guericke, *Experimenta*, 1672

“...tiny parts which are small enough break away here and there and fly into the air. In the same way, the dust flies up propelled by the feet of a passer-by”.

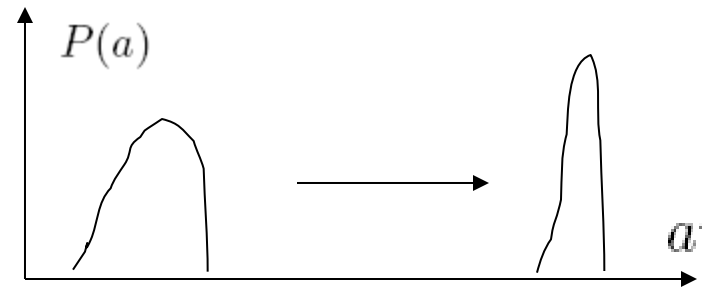
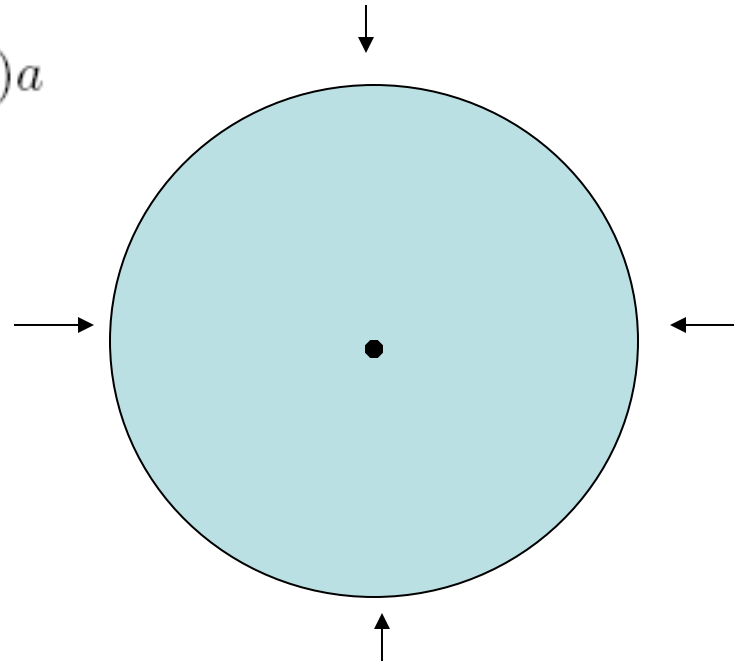
Descartes, *Les meteoros*, 1770

“... the particles of permanent air are grosser than those of vapors, a moist atmosphere is lighter than a dry one, quantity for quantity.”

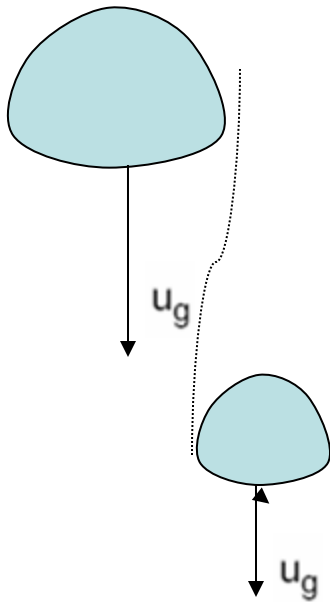
Newton, *Opticks*, 1717

$$\frac{4\pi}{3}\rho_0\frac{da^3}{dt} = \text{flux} = 4\pi\kappa(M - M_s)a$$

$$\frac{da^2}{dt} = \frac{2\kappa(M - M_s)}{\rho_0}$$



Collisions



$$u_g = g\tau$$

$$\tau = (2/9)(\rho_0/\rho)(a^2/\nu)$$

$$K_g(a, a') = \pi(a + a')^2 E(a, a') |u_g(a) - u_g(a')|$$

$$K(a, a') \simeq \pi(a + a')^2 \Delta v$$

Collision rate = target area $\pi(a + a')^2$ times velocity difference

$$K_g(a, a') = \pi(a + a')^2 E(a, a') |u_g(a) - u_g(a')|$$

How fast droplet falls?

$$6\pi R\eta_a u = mg$$

$$R = 0.01 \text{ mm} = 10 \mu\text{m}$$

$$u = \frac{2\rho_w g R^2}{9\eta_a} \simeq 1.21 \text{ cm/s}$$

$$\text{Re} \simeq 0.008$$

$$\text{Re} \propto vR \propto R^3$$

$\text{Re} \simeq 1$ already for $R = 0.05 \text{ mm}$

$$\eta_a = 1.8 \cdot 10^{-4} \text{ g/s} \cdot \text{cm}, \quad \eta_w = 0.01 \text{ g/s} \cdot \text{cm}$$

$$\rho_w = 1 \text{ g/cm}^3 \quad \rho = 1.2 \cdot 10^{-3} \text{ g/cm}^3$$

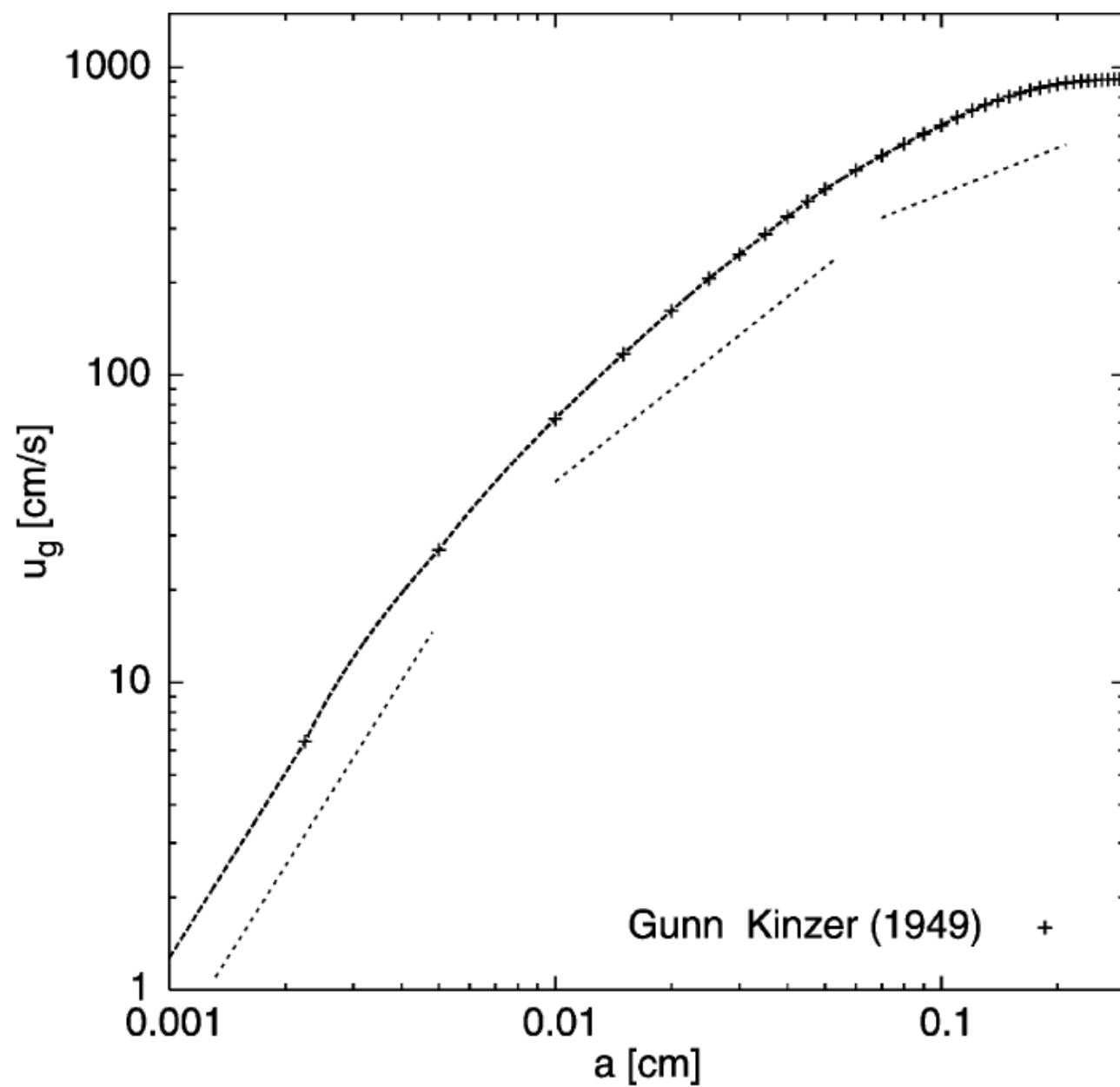


FIG. 1. Terminal fall velocity u_g as a function of cloud droplet radius a .

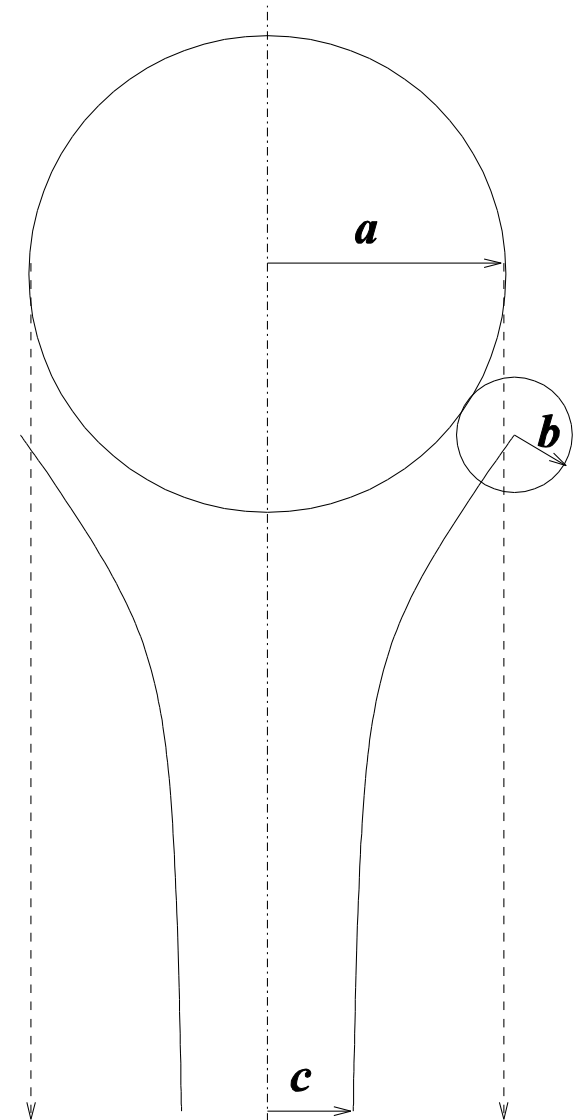
Collision efficiency is the ratio of true scattering cross-section to the geometric cross-section

$$E(a, b) = \frac{c^2}{a^2 + b^2}$$

$$E(b/a, Re) \Rightarrow c = af(b/a, Re)$$

$$\lim_{b/a \rightarrow 0, Re \rightarrow 0} E(b, a, Re) \approx \frac{b^2}{2a^2}$$

$$K_g(b, a, Re) \approx \pi a^2 \frac{b^2}{2a^2} u_g(a) = \pi g \frac{b^2 a^2}{9\nu} \frac{\rho_w}{\rho_a}$$



Let's gather stones together:

condensation at 2% supersaturation,
collisions at 100 droplets per cubic centimeter –
will get 100 hours before rain starts.

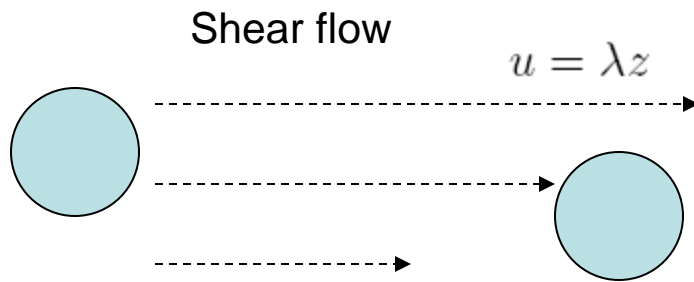
Totally unrealistic.

What did we miss?

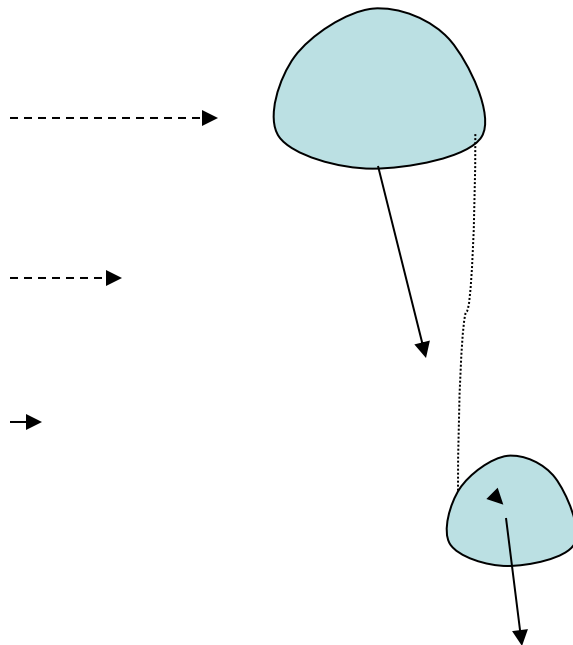
Forgot about wind

No rain without wind

No wind without turbulence



$$K_s(a, a') = \lambda \pi (a + a')^3$$



Additional collision kernel (Saffman&Turner)

Change in collision efficiency (Khain&Pinsky)

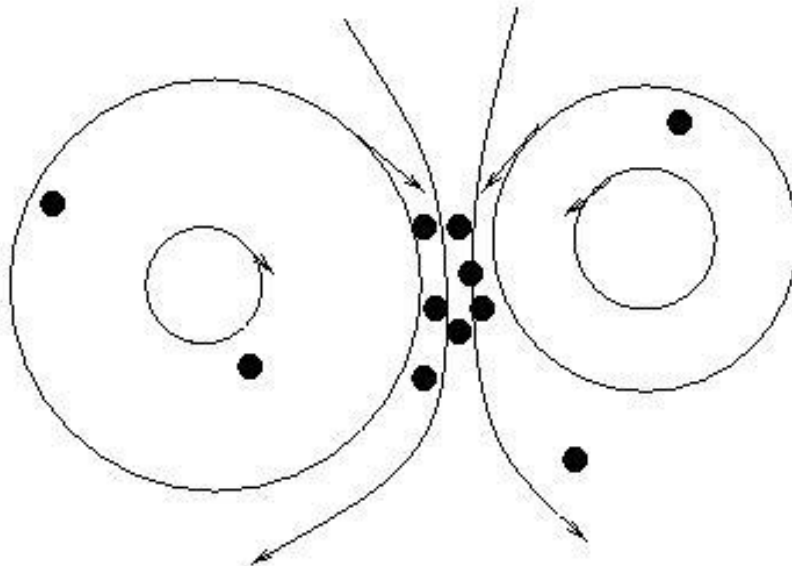
Inertial effects

$$\frac{d\mathbf{v}}{dt} = \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{v} = (\mathbf{u} - \mathbf{v})/\tau + \mathbf{g}$$

$$\mathbf{v} = \mathbf{u} + \mathbf{g}\tau - \tau[\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u}]$$

$$\nabla \cdot \mathbf{v} = -\tau \nabla(\mathbf{u} \cdot \nabla) \mathbf{u} = \tau(\Omega^2 - S^2)$$

$$\tau = (2/9)(\rho_0/\rho)(a^2/\nu)$$



Preferential concentration

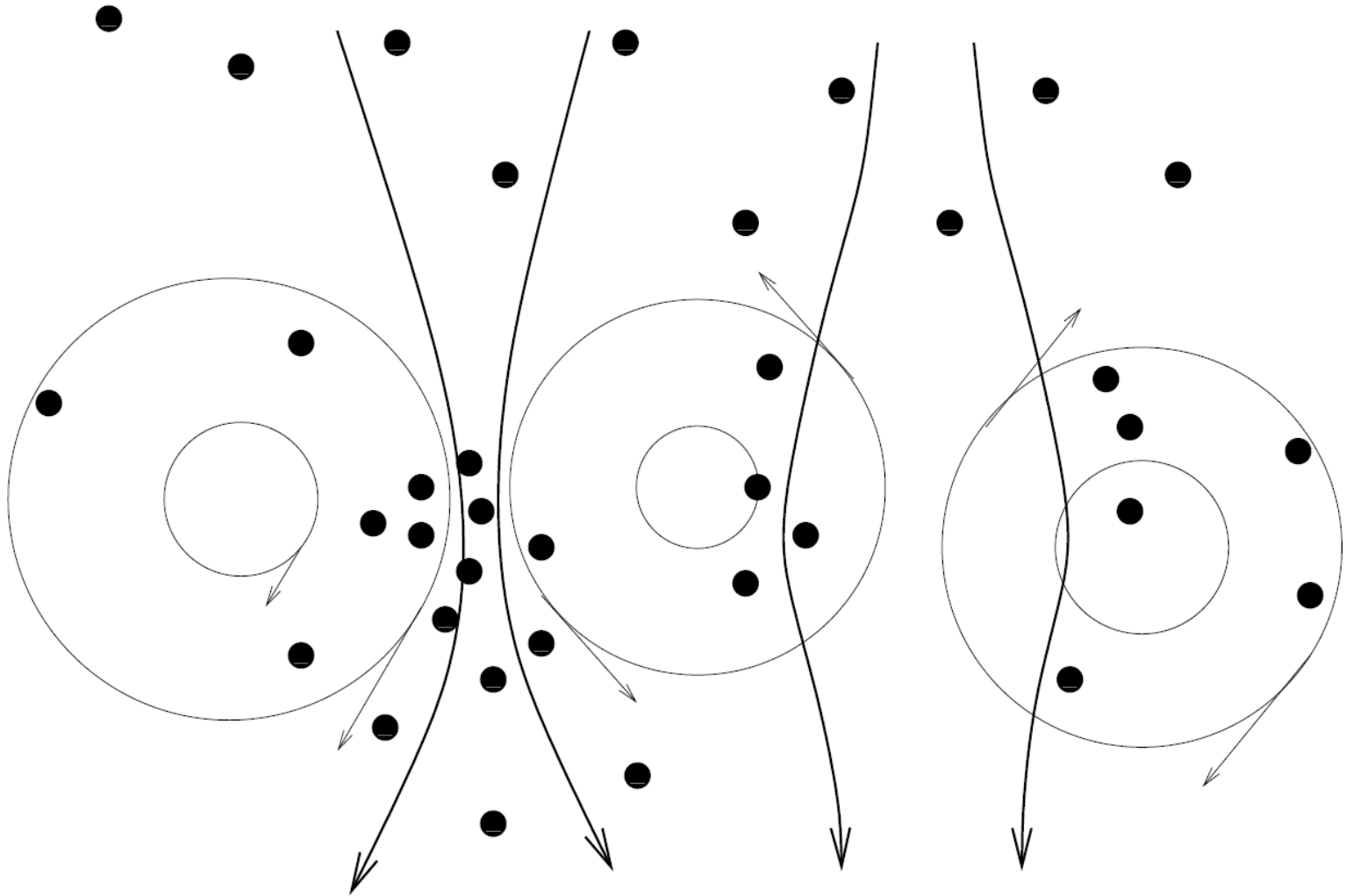
Descartes

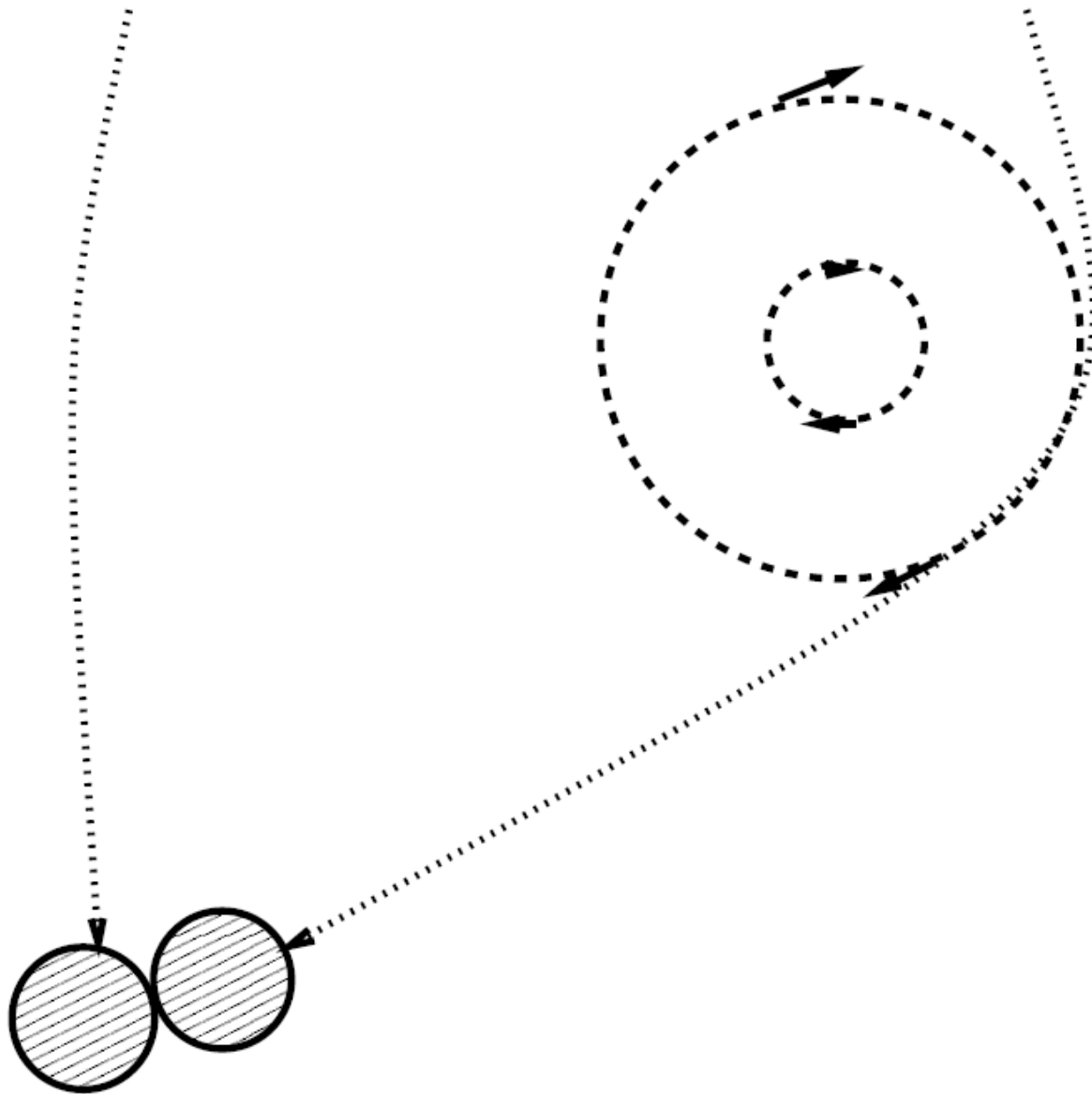
Maxey, *J. Fluid Mech*, 1987

Sling effect

Falkovich, Fouxon, Stepanov, *Nature*, 2002

Turbulence can increase mean settling velocity

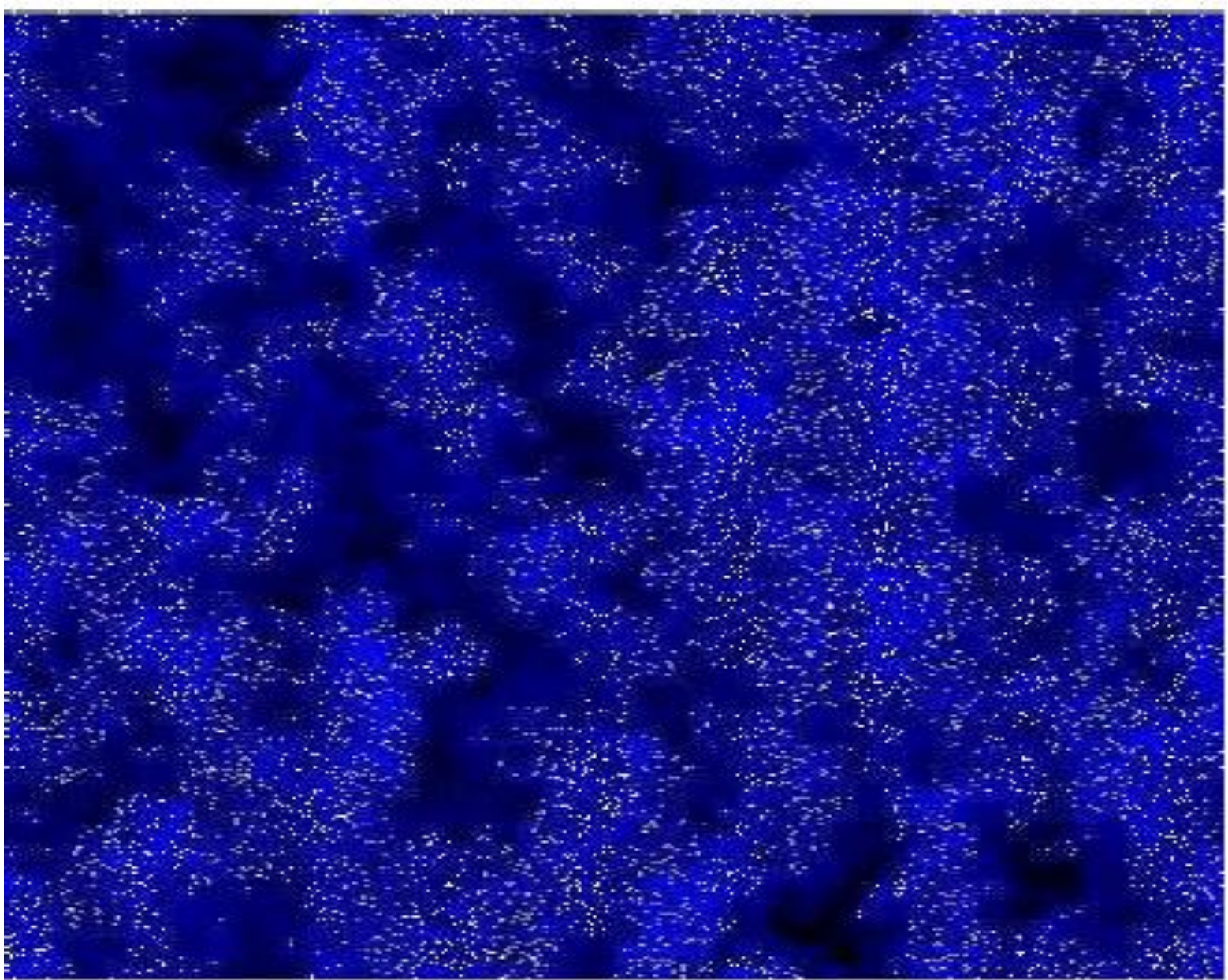




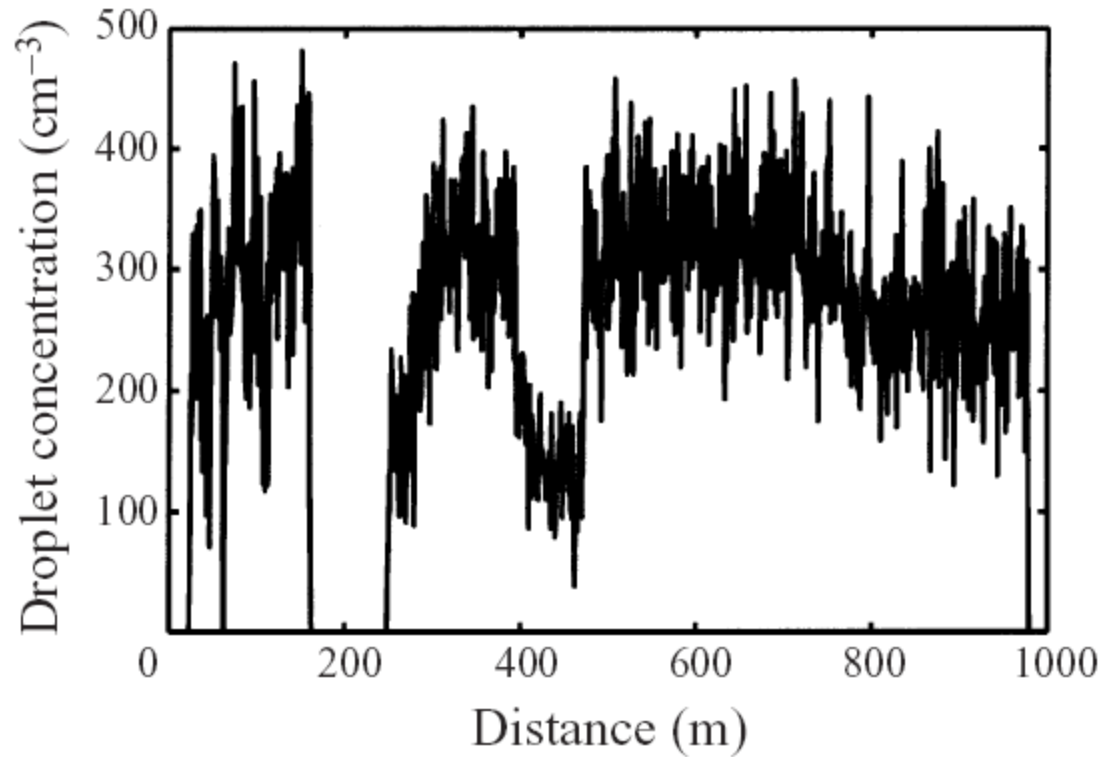
Sling effect: distant vortex causes collisions of droplets



Distribution of water vapor on the boundary between wet and dry regions



Spatial distribution of droplets (white points) and supersaturation field (blue).

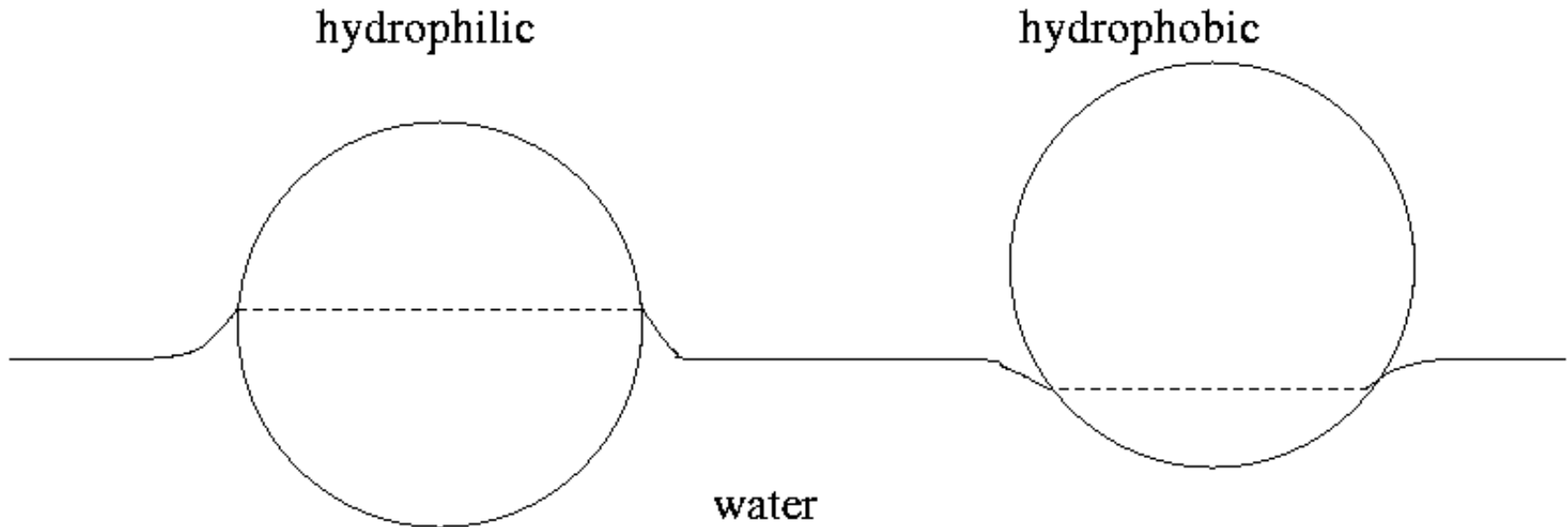


Droplet concentration in a single traverse of a cumulus cloud

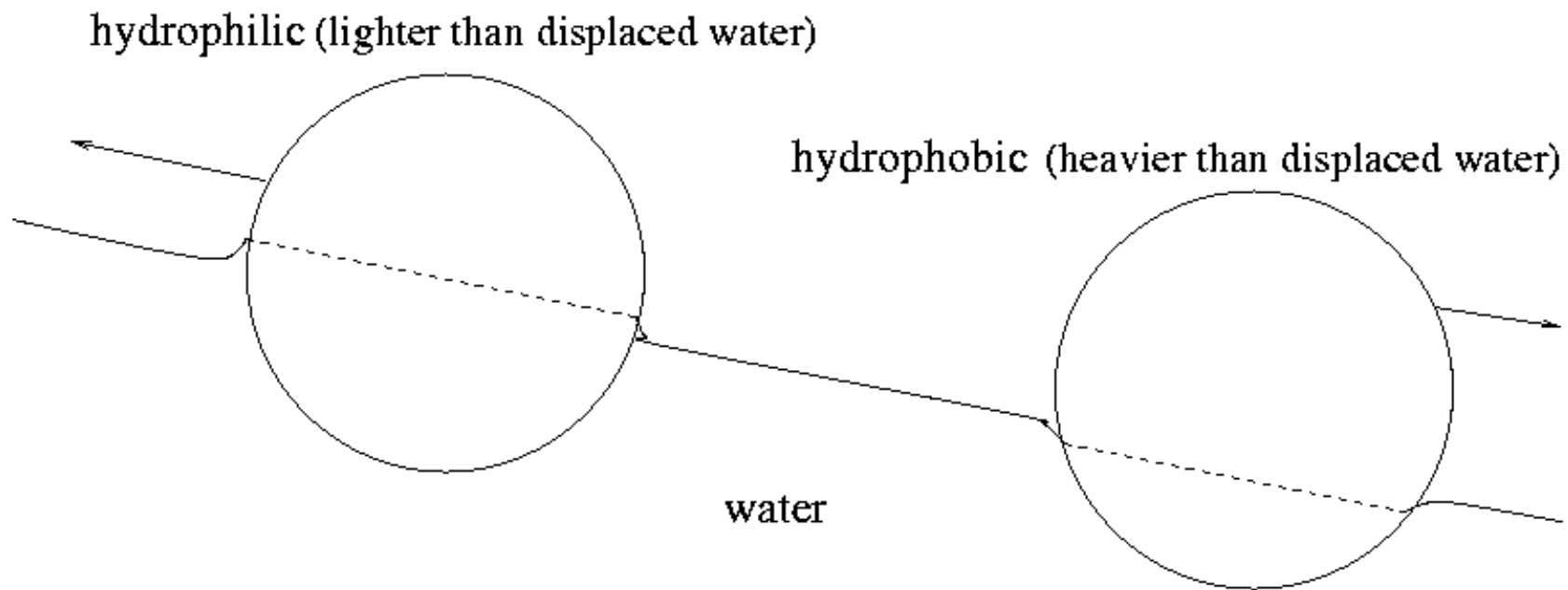
How to model that in a lab?

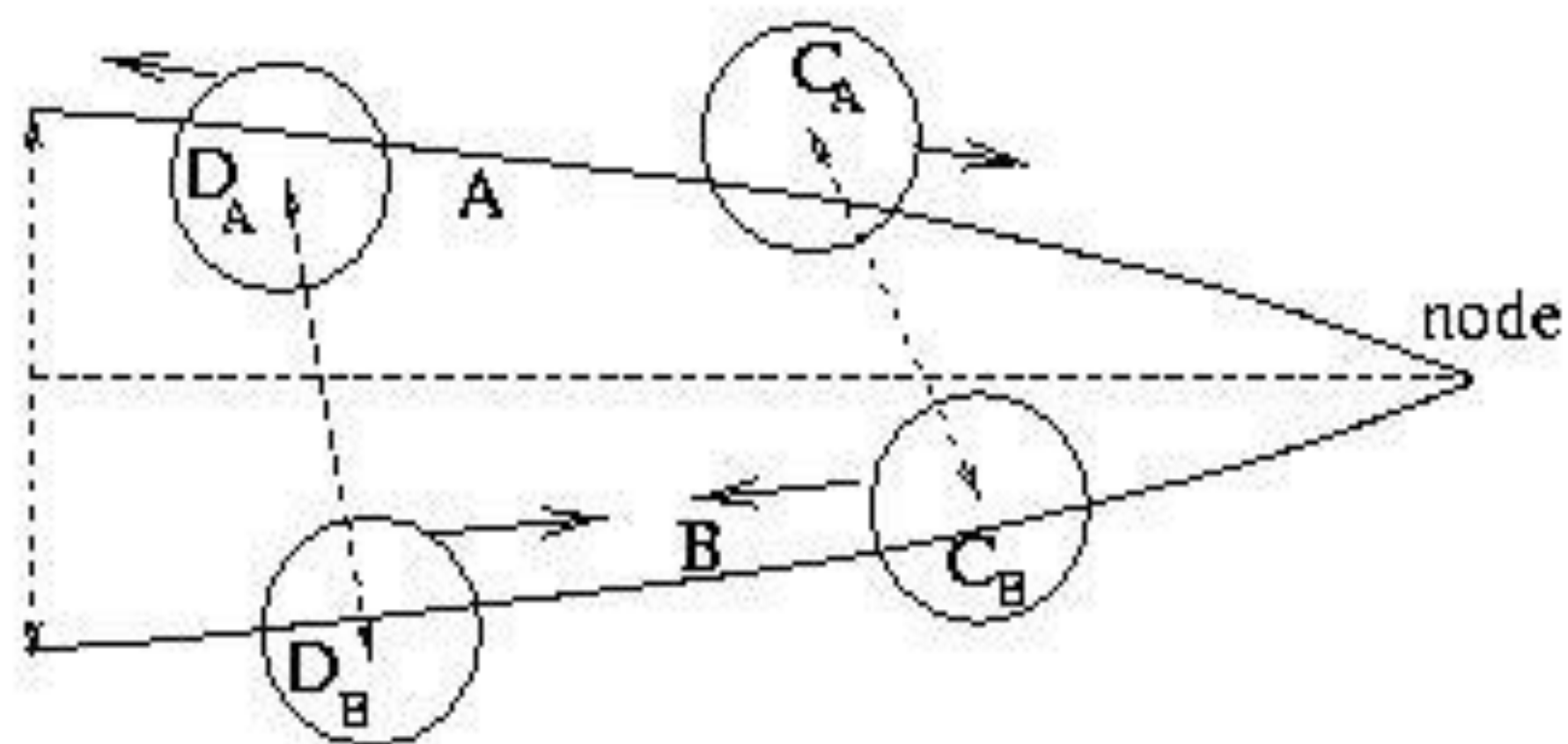
Capillarity breaks Archimedes' law

two bodies of the same weight displace different amount of water depending on their material



Motion along the inclined water surface





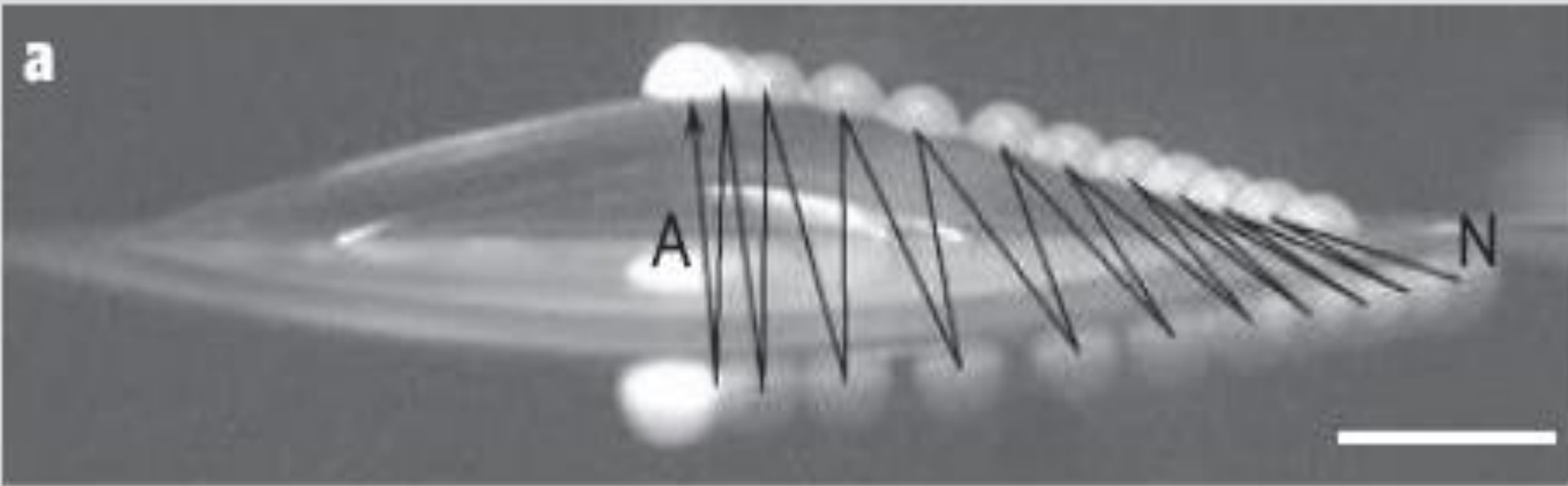
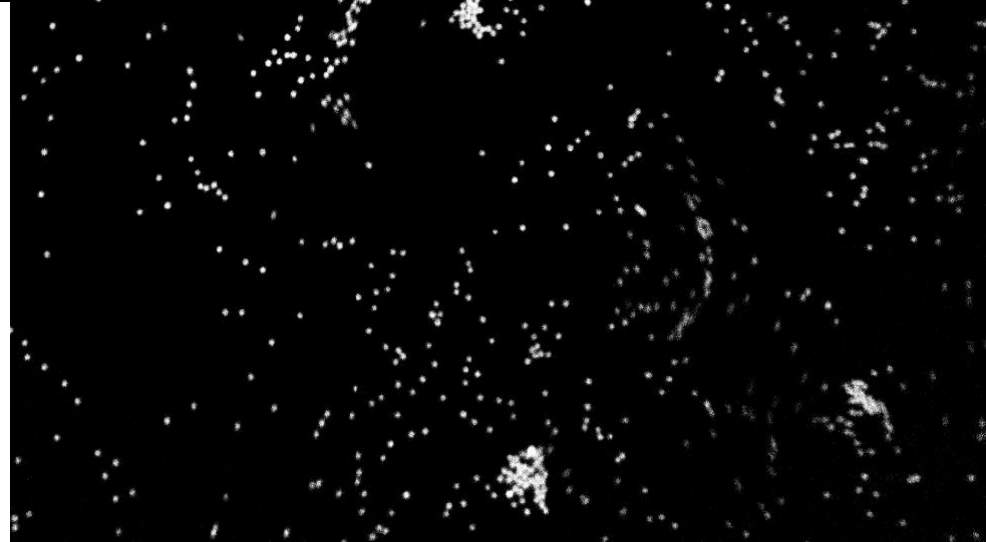
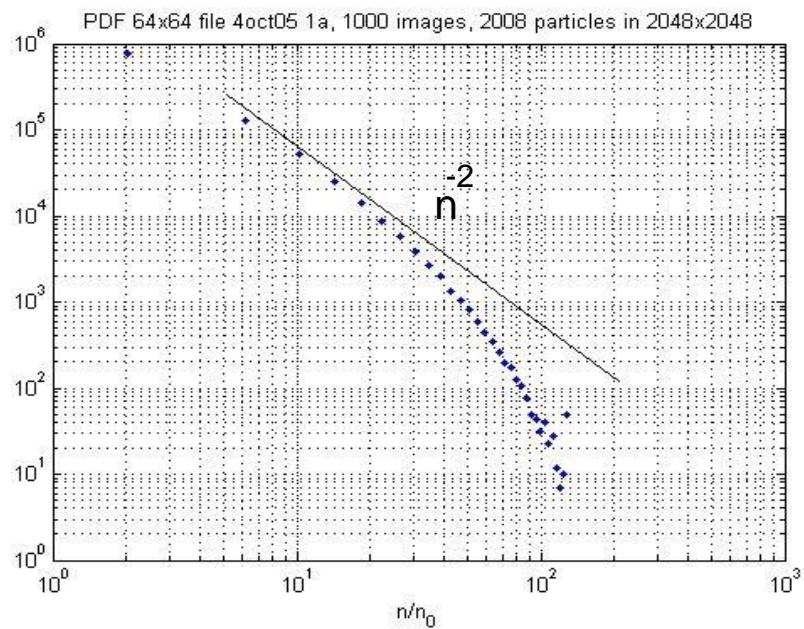
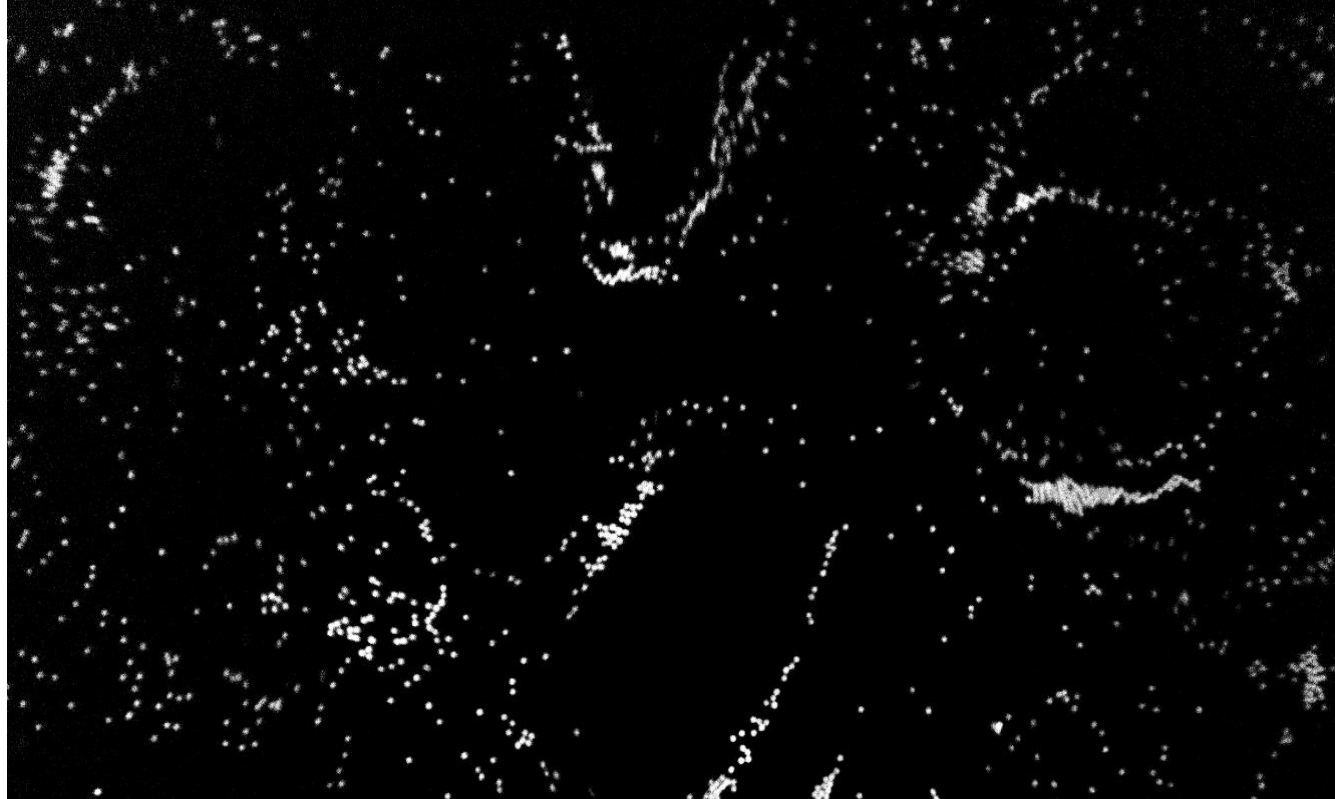


Figure 1 | Floaters in a standing wave. a, Stroboscopic picture (side view) of the drift of a hydrophobic teflon ball towards the point of maximum vertical displacement (antinode, A). Arrow shows the direction of the motion, which starts at the point of zero displacement (node, N). The line segments connect the centres of the ball positions at successive half-periods of the wave.



Falkovich, Lukaschuk, Denissenko, *Nature* 2005

Conclusion

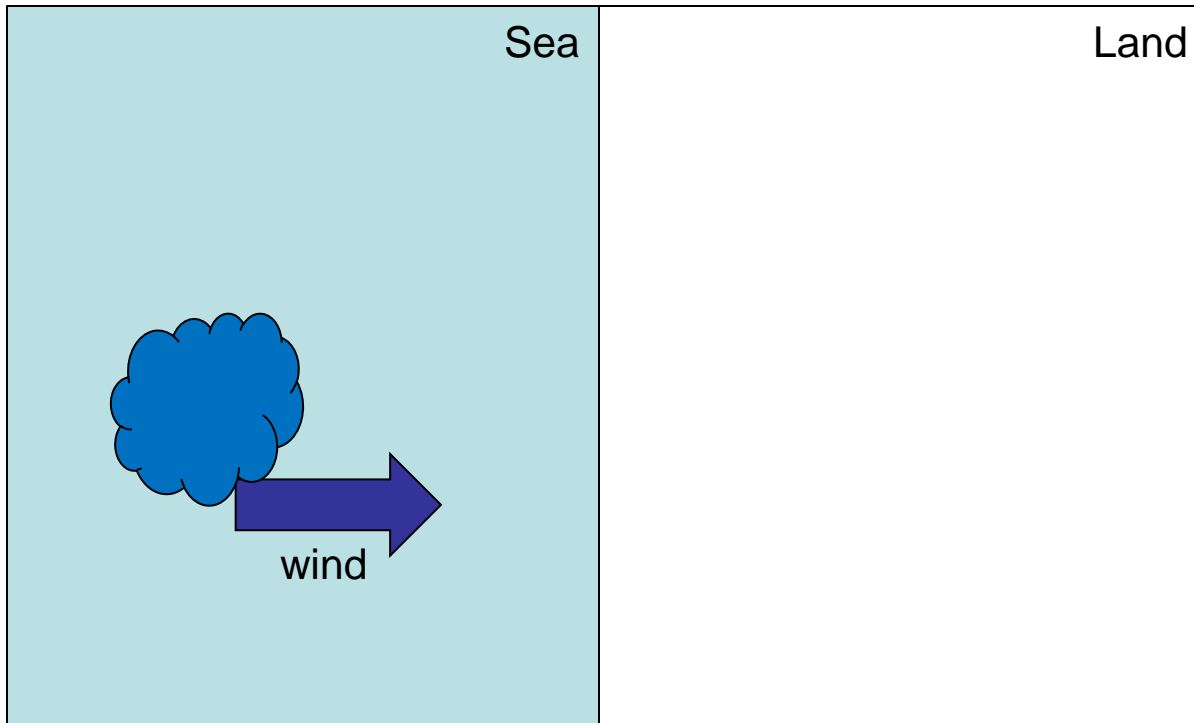
To understand and describe cloud formation and rain initiation one needs to work out (among other things)

- Theory of turbulent mixing of interacting fields
- Theory of (multi)-fractal measures
- Statistical theory of explosive events

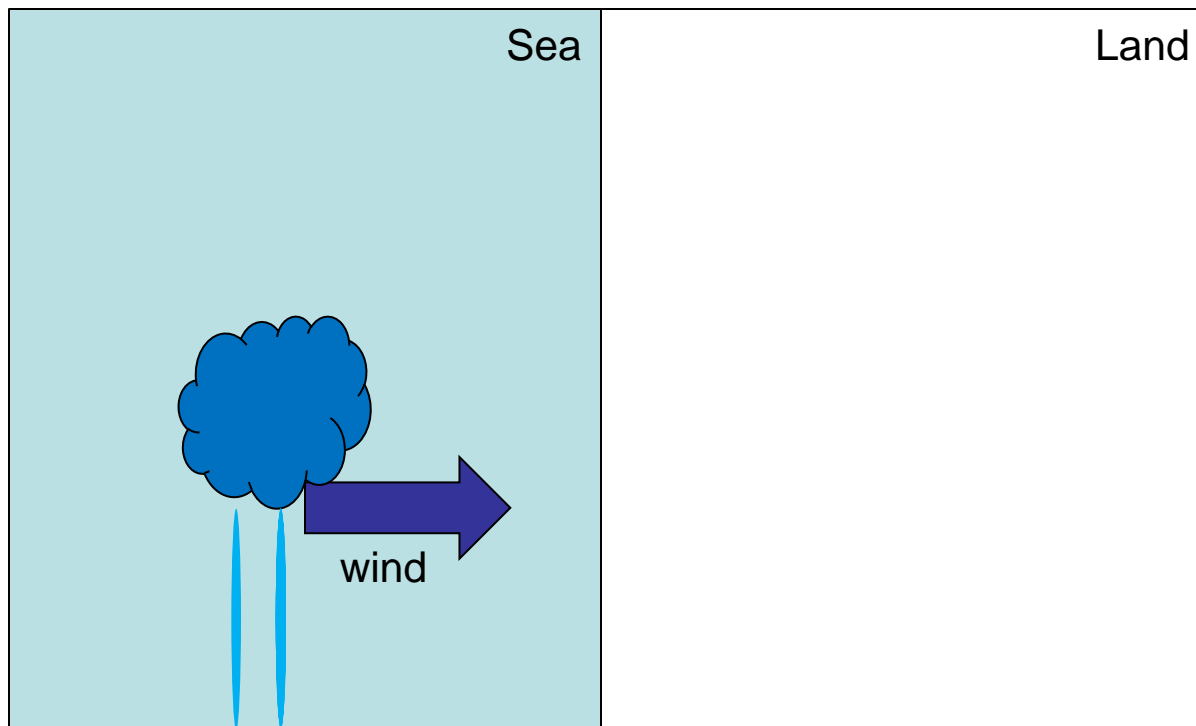
Куди йде дощ?

Where rain goes?

Winter: warm sea, cold land •

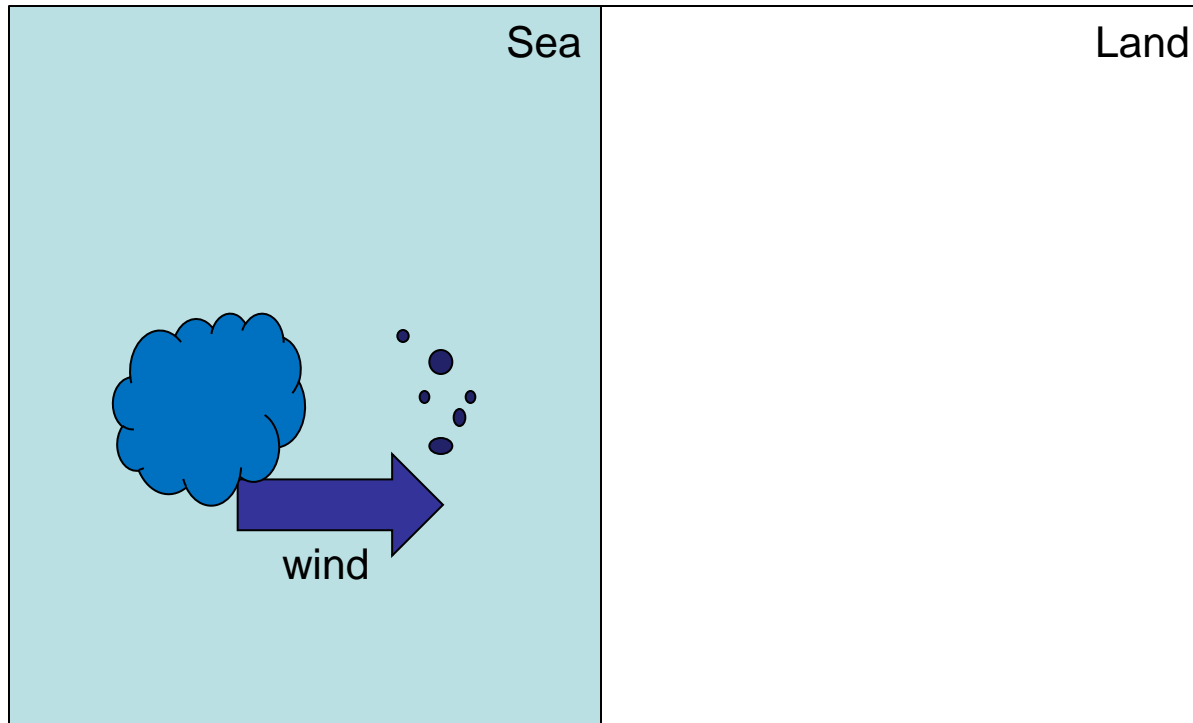


Время года - зима •



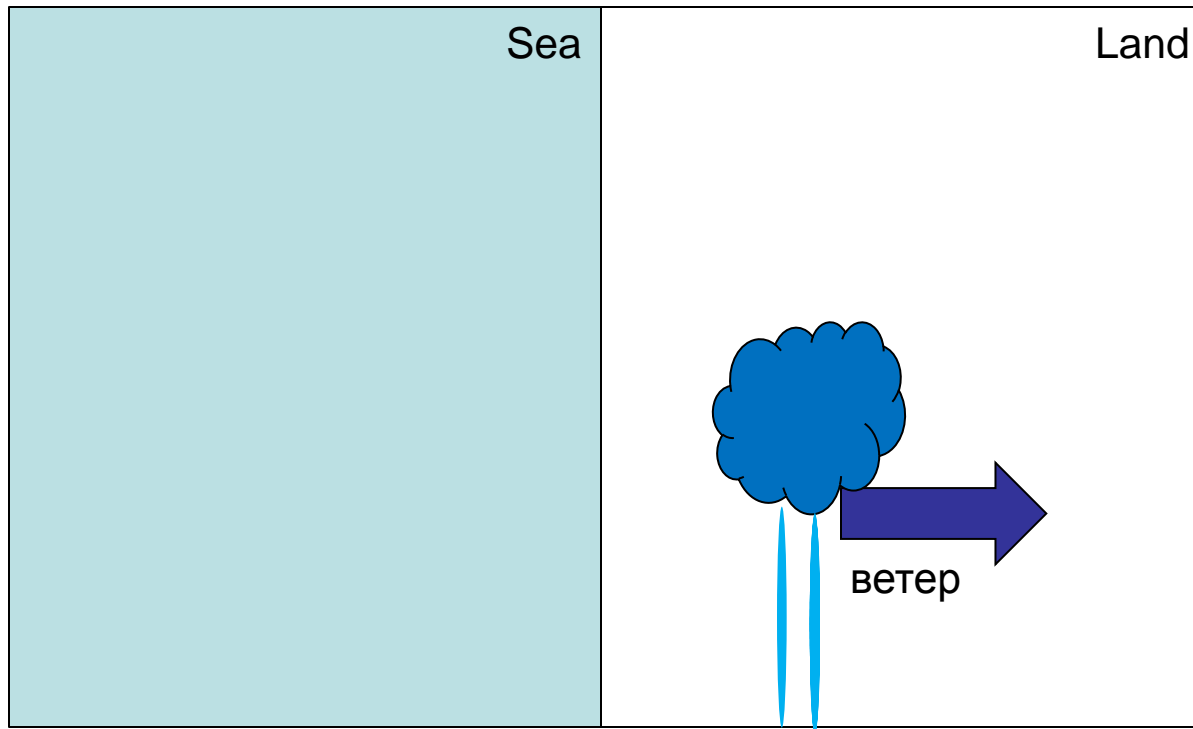
Let's add to the air a bit of dust or smoke

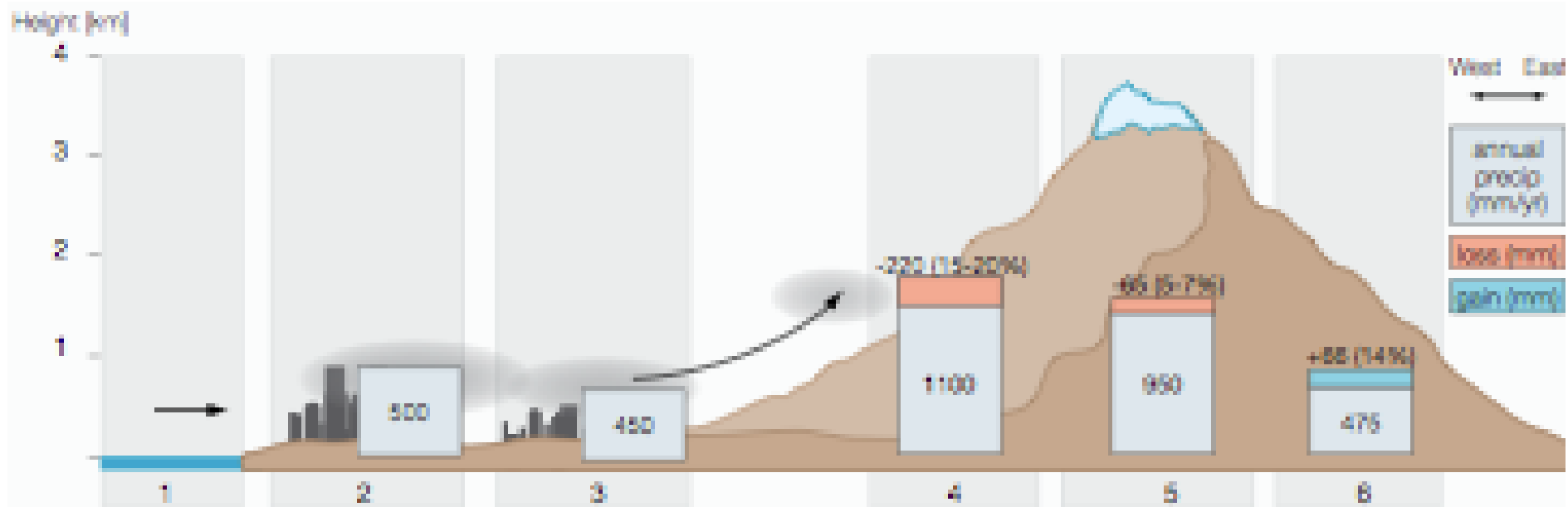
Время года - зима •

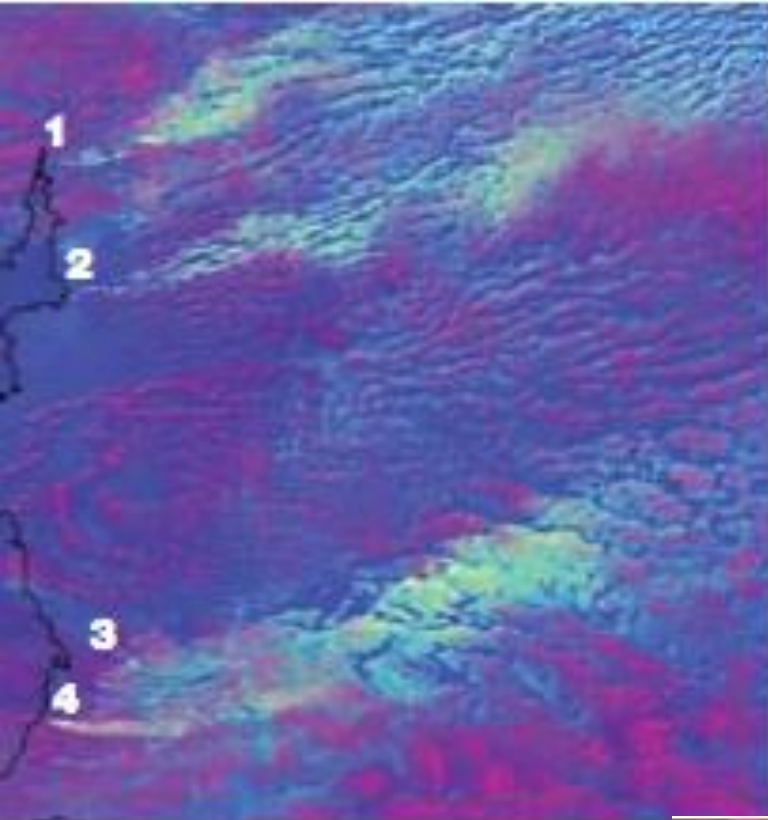


It postpones the beginning of rain

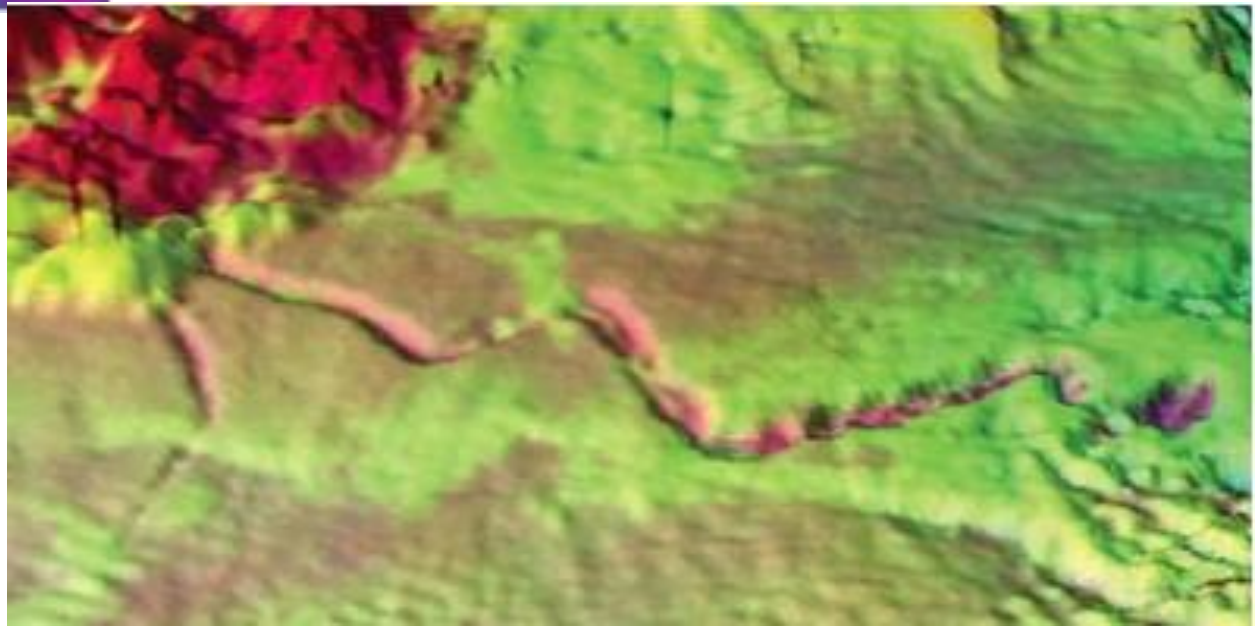
Время года - зима •







Opposite effects of air pollution (left)
and cloud seeding (right). D. Rosenberg, HU



Fluid Mechanics

The multi-disciplinary field of fluid mechanics is one of the most actively developing fields of physics, mathematics and engineering. In this book, the fundamental ideas of fluid mechanics are presented from a physics perspective.

Using examples taken from everyday life, from hydraulic jumps in a kitchen sink to Kelvin–Helmholtz instabilities in clouds, the book provides readers with a better understanding of the world around them. It teaches the art of fluid-mechanical estimates and shows how the ideas and methods developed to study the mechanics of fluids are used to analyse other systems with many degrees of freedom in statistical physics and field theory.

Aimed at undergraduate and graduate students, the book assumes no prior knowledge of the subject and only a basic understanding of vector calculus and analysis. It contains 32 exercises of varying difficulties, from simple estimates to elaborate calculations, with detailed solutions to help readers understand fluid mechanics.

Gregory Falkovich is a Professor in the Department of Physics of Complex Systems, Weizmann Institute of Science. He has researched in plasma, condensed matter, fluid mechanics, statistical and mathematical physics and cloud physics and meteorology, and has won several awards for his work.

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FALKOVICH
Fluid Mechanics

Fluid Mechanics

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